Eur. Phys. J. B **60**, 89–95 (2007)
DOI: 10.1140/epjb/e2007-00324-y

THE EUROPEAN PHYSICAL JOURNAL B

Optimal synchronizability of networks

B. Wang[1,a], T. Zhou[2], Z.L. Xiu[3], and B.J. Kim[1,b]

[1] Department of Physics, BK21 Physics Research Division, and Institute of Basic Science, Sungkyunkwan University, Suwon 440-746, Korea
[2] Department of Modern Physics and Nonlinear Science Center, University of Science and Technology of China, Hefei Anhui, 230026, P.R. China
[3] School of Environmental and Biological Science and Technology, Dalian University of Technology, Dalian Liaoning, 116024, P.R. China

Received 16 April 2007 / Received in final form 4 October 2007
Published online 29 November 2007 –

Abstract. We numerically investigate how to enhance synchronizability of coupled identical oscillators in complex networks with research focus on the roles of the high level of clustering for a given heterogeneity in the degree distribution. By using the edge-exchange method with the fixed degree sequence, we first directly maximize synchronizability measured by the eigenratio of the coupling matrix, through the use of the so-called memory tabu search algorithm developed in applied mathematics. The resulting optimal network, which turns out to be weakly disassortative, is observed to exhibit a small modularity. More importantly, it is clearly revealed that the optimally synchronizable network for a given degree sequence shows a very low level of clustering, containing much fewer small-size loops than the original network. We then use the clustering coefficient as an object function to be reduced during the edge exchanges, and find it a very efficient way to enhance synchronizability. We thus conclude that under the condition of a given degree heterogeneity, the clustering plays a very important role in the network synchronization.

PACS. 89.75.-k Complex systems – 05.45.-a Nonlinear dynamics and chaos – 05.45.Xt Synchronization; coupled oscillators

1 Introduction

Many real networked systems often exhibit common features such as the small-world effect [1] and scale-free property [2]. Network structure has a significant impact on the dynamical processes taking place on it, and particularly, the synchronization of individual elements coupled through the network structure has drawn much interest [3]. It has been shown that the ability of a network to synchronize is generally improved in both small-world networks and scale-free networks as compared to regular graphs [4–6]. However, there has been still on-going discussion on the questions of which (and how) structural property greatly affects synchronizability.

Some previous works have shown that the characteristic path length ℓ is one of the key factors: The smaller ℓ, the better synchronizability [7–9]. On the other hand, intensive research focus has been put on the role of the degree heterogeneity on synchronizability: The more homogeneous a network is (although it may have a longer ℓ), the better synchronizability can emerge [10–13]. Although the heterogeneity of networks can also be defined in terms of the spread of coupling strengths [11], we here use the term "heterogeneity" as a measure of how broad the degree distribution is. In addition, some recent works have demonstrated that disassortative networks synchronize better than assortative ones [14], and the increasing clustering hinders the global synchronization to emerge [15,16]. The majority of previous works have been based on the simulation results allowing tuning only one or a few topological measures (see also some recently proposed analytical approaches [17–19]). If all the topological properties can simultaneously vary, what will happen? Should a network with better synchronizability have some particular properties? Besides the simulated and analytical approaches, a potential way to investigate the relation between structural and dynamical properties is to track the optimization process, which will lead to some networks with specific dynamical characters [20–22]. Concerning with network synchronizability, a pioneering work [23], based on a modified simulated annealing

[a] *Present address*: Aihara Complexity Modelling Project, ERATO, JST, Institute of Industrial Science, The University of Tokyo, Tokyo 153-8505, Japan.
[b] e-mail: beomjun@skku.edu

algorithm, suggested that the network having the best synchronizability should be extremely homogeneous. In Donetti-Hurtado-Muñoz approach [23], the rewiring operation is used, thus for different initial configurations, the optimization process will lead to the same optimal result, named *Entangled Network*.

In the present paper, we aim to optimize the synchronizability of the network with the degree of each node preserved, which can be realized by edge-exchange operation [24–26]. After the achievement of the optimally synchronizable network structure for a given degree heterogeneity. We also simply minimize the clustering coefficient of the network, finding that both procedures yield the almost the same level of synchronizability. From these findings, we suggest that the clustering coefficient is the one of the most practically convenient properties to influence network synchronizability for a given degree heterogeneity.

This article is organized as follows: The concept of synchronizability and the optimization algorithm are briefly introduced in Sections 2 and 3, respectively. In Section 4, we provide main simulation results from the optimization of synchronizability for both homogeneous and heterogeneous networks, whereas Section 5 is devoted to the results from the optimization of the clustering property. Finally Section 6 is for the summary of the present work.

2 Synchronizability

Consider a network of N oscillators described by the equations of motion

$$\dot{x}_i = F(x_i) - \sigma \sum_{j=1}^{N} L_{ij} H(x_j), \tag{1}$$

where $\dot{x}_i = F(x_i)$ governs the dynamics of individual oscillator without couplings, $H(x)$ describes the interaction function between two oscillators, and σ is the coupling strength. The $N \times N$ Laplacian matrix L has elements given by

$$L_{ij} = \begin{cases} k_i, \text{ for } i = j, \\ -1, \text{ for } j \in \Lambda_i, \\ 0, \text{ otherwise}, \end{cases} \tag{2}$$

where Λ_i denotes the set of i's neighbors and k_i is the degree of the vertex i. The Laplacian matrix plays a very important role in the study of the synchronization of identical oscillators. All the eigenvalues of the Laplacian matrix L are known to be real positive and the smallest eigenvalue λ_1 is zero due to the vanishing row sum of L, i.e., $0 = \lambda_1 \leq \lambda_2 \ldots \leq \lambda_N$. Network synchronizability is well quantified by the ratio R of the the largest and the smallest nonzero eigenvalues, i.e., $R \equiv \lambda_N/\lambda_2$ [27,28], and the network is synchronizable if $R < \beta$ with β being a some constant depending on F and H. Consequently, the smaller the eigenratio R, the easier it is to synchronize the oscillators, and vice versa. This approach based on the linear stability of perfectly synchronized oscillators is very useful in a practical point of view, since R depends only on the topology of interactions among oscillators. Having reduced the problem of optimizing network synchronizability to the one of finding the smallest eigenratio R, we below present our numerical method to attain the best possible synchronizability for a network with a given degree sequence.

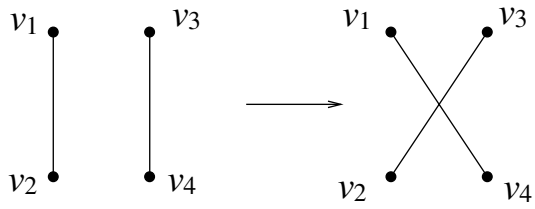

Fig. 1. The edge-exchange operation during optimization procedure. Two edges are chosen randomly and the two vertices at the ends of edges are exchanged with each other. Multiple edges and self-edges are prohibited.

3 Algorithm: Edge exchange and memory tabu search

As shown in Figure 1, the procedure of the edge-exchange operation in an undirected network goes as follows: (i) Randomly pick two existing edges $e_1 = (v_1, v_2)$ and $e_2 = (v_3, v_4)$, with all four vertices (v_1, v_2, v_3, v_4) being different. (ii) Exchange these two edges to obtain $e_1' = (v_1, v_4)$ and $e_2' = (v_2, v_3)$. We take the eigenratio R as the quantity to minimize, and apply the heuristic algorithm, called memory tabu search (MTS), popularly used in the area of applied mathematics and computational science [29], as follows:

1. At the initial stage at time $k = 0$, we generate the so-called tabu list $T_{k=0} = \{G_0^{(1)}, G_0^{(2)}, \cdots, G_0^{(n)}\}$ composed of n graphs, where the subscript '0' refers the present time ($k = 0$), and each element in T_0 is a graph of the size N randomly generated for a given degree distribution. We compute eigenratios for all graphs in the tabu list and pick the graph $G_k^{(m)}$ ($m \in [1, n]$) which has the lowest eigenratio $R_k^{(m)}$. We also set $G^* = G_k^{(m)}$ and $R^* = R_k^{(m)}$ to store the graph with the lowest eigenratio found so far. Accordingly, one may regard $G_k^{(m)}$ as the locally optimal graph within T_k, and G^* is the globally optimal one found so far.
2. From $G_k^{(m)}$, the locally optimal graph for the given tabu list at time k, obtain the trial graph G' by using the edge exchange method shown in Figure 1 for two randomly selected edges of $G_k^{(m)}$, and compute $R' = R(G')$. If the trial graph has a better synchronizability than $G_k^{(m)}$, i.e., if $R' \leq R_k^{(m)}$, we always accept the try and update the tabu list by inserting G' to the list. Even when the inequality is not satisfied, or the trial graph is not better than $G_k^{(m)}$, we accept the try if $|R' - R_k^{(j)}|/R' < \delta_j$ ($j = 1, 2, \cdots, n$), with δ_j a random

number uniformly distributed in [0.50,0.75] (Different choice of δ_j does not make much difference). If the trial graph is rejected, we do not insert G' into the tabu list. We keep the size of the tabu list fixed, and thus when a graph is newly inserted, we apply first-in-first-out method, and remove the graph which entered the list at the earliest time. Update G^* and R^* if newly found graph has the lowest eigenratio. Increase the time k and repeat the procedure.

In the viewpoint of statistical mechanics, the above described MTS method is very similar to the Monte-Carlo (MC) algorithm. However, the use of the list to keep n recently found local optima presumably make the optimization search algorithm quite efficient, since in the standard MC method, we only keep the last configuration. The probabilistic acceptance through the use of δ, although heuristic, is also similar to the MC method at finite temperatures.

4 Results: Optimizing synchronizability

As initial network structures to begin with, we take the scale-free network proposed by Barabási and Albert (BA) [2], the scale-free network with tunable clustering by Holme and Kim (HK) [30], the small-world network by Watts and Strogatz (WS) [1], the Erdös and Rényi (ER) random network [31], and the regular network. It is to be noted that the former two, BA and HK, in the above list have heterogeneous degree distributions, while the latter three have homogeneous ones. We find that the simulation results for the ER network and the regular network are in accord with the main conclusions (see below) drawn from the other three networks, and henceforth present only results for the WS, the BA, and the HK networks.

4.1 Watts-Strogatz network

We first investigate the optimization of synchronizability for the WS network as an initial graph. Figure 2 reports the eigenratio R versus the iteration step obtained for the WS network at the rewiring probability $p = 0.1$ (see Ref. [1] for details). One can see that the optimization method of the MTS works efficiently and the eigenratio is reduced dramatically. In Figure 3, we plot topological measures such as the characteristic path length ℓ, the clustering coefficient C [1], the assortativity r [32], and the modularity M defined as [33,34]

$$M \equiv \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j \neq i} S_{ij}, \tag{3}$$

where S_{ij} is the number of common neighbors of nodes i and j divided by their total number of neighbors. It is clearly seen that the optimization of synchronizability is accompanied by the changes of the structural and topological network properties: the characteristic path length becomes shorter, the assortativity changes from positive to

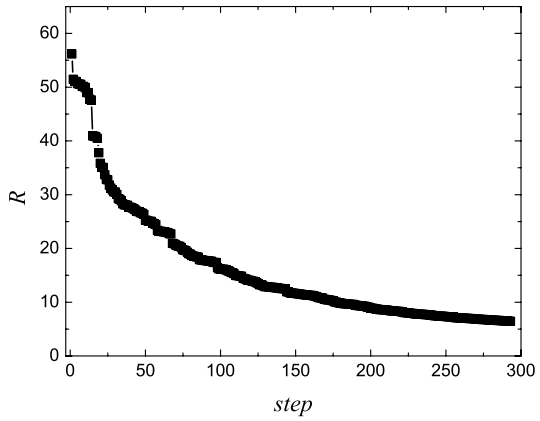

Fig. 2. The eigenratio R for the WS network of the size $N = 400$ with the average degree $\langle k \rangle = 6$ at the rewiring probability $p = 0.1$ is shown to decrease significantly as the optimization procedure in Section 3 is repeated. Only the iteration step at which R is reduced are recorded and used for the horizontal axis.

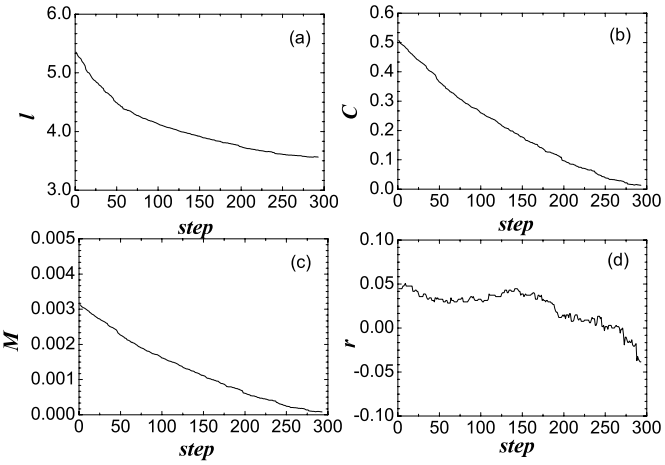

Fig. 3. During the MTS optimization of synchronizability for the WS network (see Fig. 2), (a) the characteristic path length ℓ, (b) the average clustering coefficient C, (c) the modularity M, and (d) the assortativity r are measured.

negative, indicating that degrees of neighbors become, although weak, negatively correlated. Furthermore, the clustering coefficient is reduced monotonically, approaching zero. The lower degree of clustering and disassortativity in the optimally synchronizable networks are consistent with existing studies [14–16]. More interestingly, the modularity also decreases, in accord with a recent work [35], which implies that a network with strong modular structures can be more difficult to synchronize.

It has been found that many biological and technological networks contain motifs, that is, some specific subgraphs appearing much more frequently than that observed in random graphs with the same degree sequence [36,37]. Loop is one of the simplest but most significant subgraphs, for it accounts for the multiplicity of paths between any two nodes [38]. In Figure 4, we show, via the measurements of the numbers of loops of the

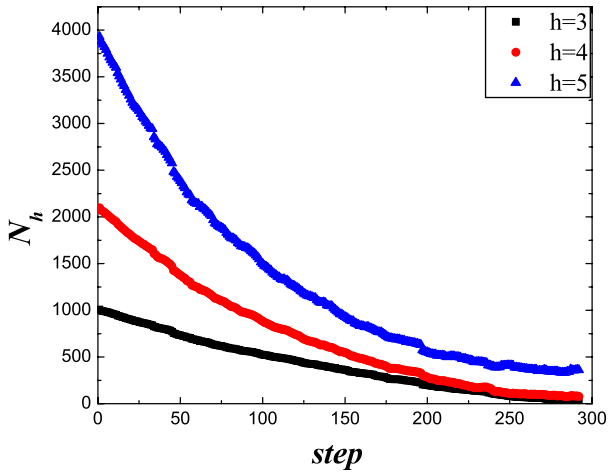

Fig. 4. (Color online) Number of h-loops, N_h, in the network vs. iteration steps of the MTS optimization of synchronizability. The value of N_h in step zero corresponds to the case of initial network, which is a WS network with $N = 400$, $\langle k \rangle = 6$ and $p = 0.1$.

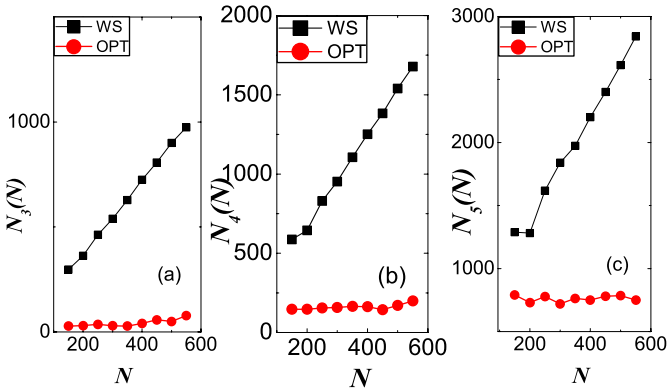

Fig. 5. (Color online) Number of h-loops, $N_h(N)$, vs. N in WS networks (WS, black squares) and the corresponding optimal networks obtained from the MTS optimization of synchronizability (OPT, red circles), respectively. The network sizes from $N = 150$ up to $N = 550$ are used. In all those simulations, the rewiring probability and average degree are $p = 0.1$ and $\langle k \rangle = 6$. The three panels from left to right are for (a) $h = 3$, (b) $h = 4$, and (c) $h = 5$, respectively.

sizes 3, 4, and 5, the change of the loop structure during the optimization process. Due to the practical limitation on computational capability, we did not calculate loops bigger than the size 5. One can observe that the number of loops drops drastically during the optimization process, which indicates that the dense loops may hinder the global synchronization. As shown in Figure 5, in WS networks, the number of loops increases linearly with the network size. In contrast, the numbers of loops in the corresponding optimal networks stay at almost the same levels, indicating the vanishing densities of loops as N becomes larger.

All these results indeed state that the optimal networks belong to a class of networks in which there are few number of loops, different from the majority of real

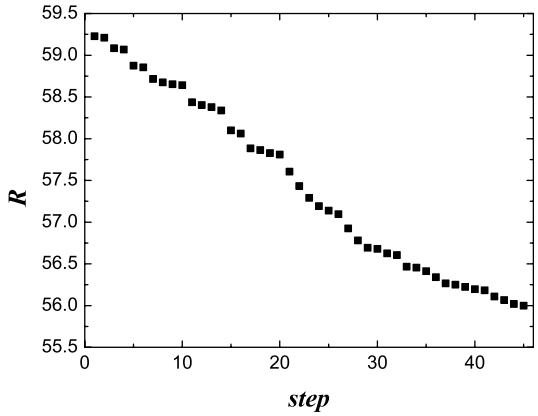

Fig. 6. The eigenratio R vs. iteration steps of a BA network with $N = 400$ and $\langle k \rangle = 6$ for the MTS optimization of synchronizability. Only the steps in which R being reduced are recorded.

biological and technological networks. One of the possible reasons for the discrepancy is that the optimally synchronizable network structure may have other disadvantages, such as the vulnerability to structural perturbations, the increase of the cost to build networks [39], and so on.

4.2 Barabási-Albert network

As shown above, the MTS algorithm appears to be very effective for homogeneous networks: The eigenratio of WS networks is reduced by the factor about 0.85 in comparison to the original WS network (see Fig. 2). We next apply the same MTS method for networks with heterogeneous degree distributions, i.e., the BA scale-free networks. As shown in Figure 6, our method also enhances the synchronizability of the BA network, however, the enhancement is not as big as for the WS network, only about 6% decrease of R for the BA, while it was 85% for the WS networks. Other structural properties (not shown here) such as the characteristic path length, the clustering coefficient, the modularity, and assortativity are found to show the same decreasing behaviors as for the WS network in Figure 3. However, the changes are significantly smaller than the WS network since the initial BA network already has small values for those quantities. We also measured the number of loops of the sizes $h = 3$, 4, and 5 only to find small differences between the original BA network and the optimized one.

The above findings made for the optimization of BA networks imply that it is indeed difficult to enhance synchronizability of the BA network. We next pursue the answer to the question which structural characteristics of the BA network led us to the difficulty in optimizing synchronizability. Particularly, we check below whether or not the heterogeneous degree distribution in the BA network, which is one of the striking differences from the WS network, is the cause of the inefficiency.

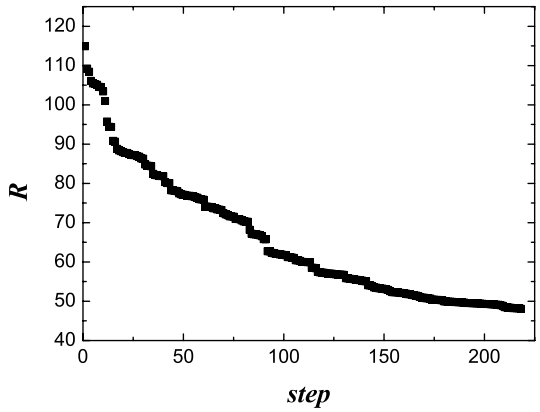

Fig. 7. The eigenratio R vs. iteration steps of a HK network with $N = 400$ and $\langle k \rangle = 6$ during the MTS optimization of synchronizability. Only the steps in which R being reduced are recorded.

4.3 Holme-Kim network

We here use the Holme-Kim model [30] of a scale-free network with the tunable clustering coefficient. We first generate the network with the heterogeneous power-law degree distribution but with the clustering coefficient much greater than the BA network. If the difficulty we faced in the optimization for the BA network described above is due to the heterogeneous degree distribution, one expects the same inefficiency also for the HK network. On the other hand, if the low clustering coefficient in the BA network is the origin, one can have different result for the HK network, i.e., much greater enhancement of synchronizability. Figure 7 displays R during the MTS optimization procedure applied to the HK network of the size $N = 400$ and $\langle k \rangle = 6$. We tune the clustering coefficient of the initial HK network to $C \approx 0.5$. As the MTS optimization of the eigenratio proceeds, it is clearly seen that one can reduce R significantly from $R \approx 110$ to 48, which corresponds to about 50% of decrease. We also measure the clustering coefficient during the optimization (not shown) and observe it decreases from $C \approx 0.5$ for the initial HK network to 0.05 monotonically. These observations imply that the reason why it is so difficult to optimize the synchronizability of the BA network is not actually due to the heterogeneity of the degree distribution but due to the very low level of the clustering: C is already too small and it is difficult to reduce it further. Consequently, we suggest that the optimization of the synchronizability of complex networks is very closely related with the clustering property of the given network. It is noteworthy that the value of R we achieve for the HK network is still much bigger than the corresponding value in Figure 2, which indicates that homogeneous network with lower clustering is the most optimal network structure.

5 Results: Optimizing clustering property

Based on observations in Section 4 that the optimization of synchronizability is closely related with the clustering property of the original network, we in this section apply an opposite way: Take the clustering coefficient, instead of the eigenratio, as the quantity to minimize, and measure R during the optimization of the clustering property. In contrast, we in the preceding section measured C during the optimization of R. The use of the clustering coefficient as the object function to optimize also has a practical efficiency since only the clustering coefficients of vertices related with the exchanged two edges need to be newly computed, which takes only $O(1)$ operations. In order to simplify further, we adopt the frequently used Monte-Carlo method at zero temperature: Define the energy as the total clustering coefficient, and use the standard Metropolis algorithm at zero temperature with the edge exchange in Figure 1 as the local MC try [25].

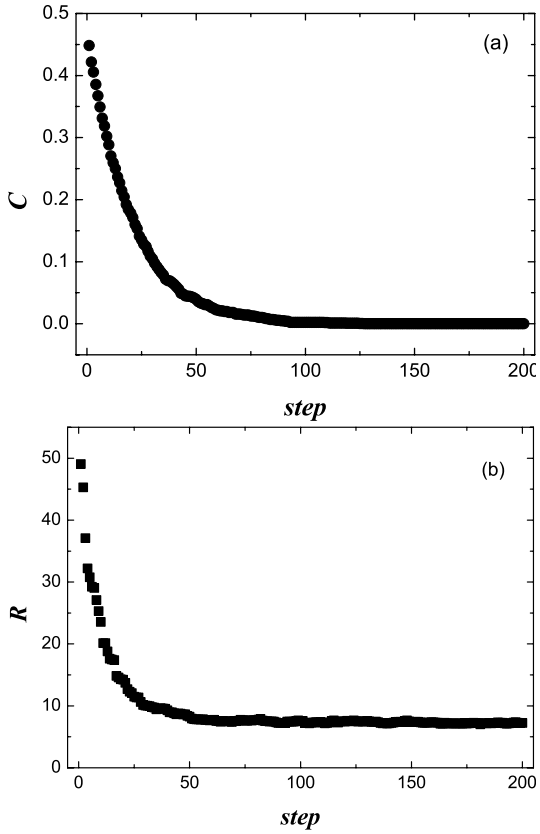

Fig. 8. (a) The clustering coefficient C and (b) the eigenratio R from the MC optimization of the clustering coefficient are shown as functions of the MC steps. The WS network of the size $N = 400$ and $\langle k \rangle = 6$ at $p = 0.1$ is used as the initial network. [Compare with Figs. 3b and 2, respectively].

In Figure 8, we summarize our results obtained from the MC optimization of the clustering coefficient for the WS network as the initial network structure. The similarities between Figures 8a and 3b, and between Figures 8b and 2, respectively, are striking. This clearly indicates that the clustering coefficient plays a very important role in the synchronizability of networks: One can achieve the same level of synchronizability either by directly optimizing synchronizability or by optimizing the clustering coefficient;

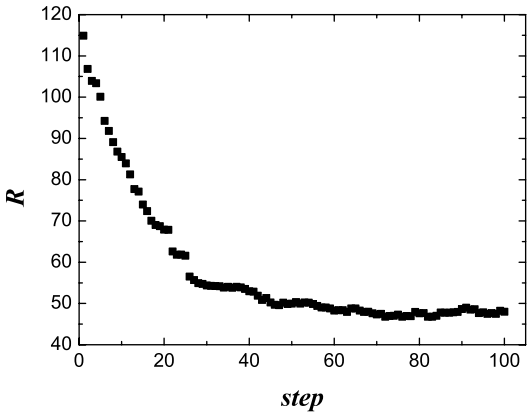

Fig. 9. The eigenratio R versus MC step for the HK network as the initial structure. The simple MC method with C taken as the energy function has been used.

the latter can be done much more efficiently than the former.

We then repeat the same MC optimization of the clustering coefficient for the BA network and also for the HK network. For the BA network (not shown here) we get results very similar to Figure 6, which is easily understood since the original BA network has already very weak clustering property and thus further optimization is almost impossible. For the HK network, we present our MC optimization result of the clustering in Figure 9. Again, we find that one can achieve optimal synchronizability by minimizing the clustering coefficient (compare with Fig. 7).

6 Conclusion and discussion

In summary, a heuristic algorithm, memory tabu search, in combination of the edge-exchange method to keep the degree of each vertex unchanged, has been used to optimize network synchronizability. For the homogeneous and heterogeneous networks, with and without a high level of clustering, topological characters have been measured, which suggests that a network with shorter average distance, lower clustering, negative degree-degree correlation and weaker modular structure is easier to synchronize. In addition, we investigated the change of loop structure in the optimization process, and found that the number of loops decreases as synchronizability is enhanced.

Since each node can only impact its neighbors [see Eq. (1)], if there is one path of length l between node i and j, then, along this path, it takes l steps transferring the synchronization signal from i to j (or from j to i). Therefore, if there are so many paths of different lengths between nodes i and j, the synchronization signal of i at a given time will arrive at j along different paths at different times, which may result in a destructive interference effect. It may be the reason why dense loops, including the loop of the size three related with the clustering coefficient, may hinder the global synchronization. An extreme case is that for directed networks, the one with the highest synchronizability (i.e. with eigenratio R being equal to 1) is a tree structure without any loops [40]. Adding one loop of length 2 [41], the eigenratio will be doubled [42,43]. However, one has to be cautious extending the conclusion in reference [40] toward the case of the nonidentical oscillators, as pointed out in reference [44]. We believe this work will be helpful for the in-depth understanding about the role of loops in network synchronization.

Many previous works have focused on synchronization in heterogeneous networks, especially in scale-free networks. A common cognition is that heterogeneity hinders the global synchronization in general. In this work, we have clearly shown that the clustering property also plays an important role when the degree heterogeneity is given and that, more importantly, one can achieve almost maximal synchronizability only by reducing the clustering coefficient. We believe that our finding is practically useful since the clustering coefficient is a local quantity and thus has a computational advantage to other global properties.

We thank P. Holme for useful comments. B.J.K. was supported by grant No. R01-2005-000-10199-0 from the Basic Research Program of the Korea Science and Engineering Foundation, T.Z. was supported by NNSFC under grant No. 10635040.

References

1. D.J. Watts, S.H. Strogatz, Nature **393**, 440 (1998)
2. A.-L. Barabási, R. Albert, Science **286**, 509 (1999)
3. R. Albert, A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002); M.E.J. Newman, SIAM Rev. **45**, 167 (2003); S.N. Dorogovtsev, F.F. Mendes, Adv. Phys. **51**, 1079 (2002); S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, D.-U. Hwang, Phys. Rep. **424**, 175 (2006)
4. L.F. Lago-Fernndez, R. Huerta, F. Corbacho, J.A. Siguenza, Phys. Rev. Lett. **84**, 2758 (2000)
5. P.M. Gade, C.-K. Hu, Phys. Rev. E **62**, 6409 (2000)
6. X.F. Wang, G. Chen, Int. J. Bifurcation & Chaos **12**, 187 (2002)
7. M. Barahona, L.M. Pecora, Phys. Rev. Lett. **89**, 054101 (2002)
8. P.G. Lind, J.A.C. Gallas, H.J. Herrmann, Phys. Rev. E **70**, 056207 (2004)
9. T. Zhou, M. Zhao, B.-H. Wang, Phys. Rev. E **73**, 037101 (2006)
10. T. Nishikawa, A.E. Motter, Y.-C. Lai, F.C. Hoppensteadt, Phys. Rev. Lett. **91**, 014101 (2003)
11. A.E. Motter, C.S. Zhou, J. Kurths, Europhys. Lett. **69**, 334 (2005)
12. H. Hong, B.J. Kim, M.Y. Choi, H. Park, Phys. Rev. E **69**, 067105 (2004)
13. M. Zhao, T. Zhou, B.-H. Wang, W.-X. Wang, Phys. Rev. E **72**, 057102 (2005)
14. M. di Bernardo, F. Garofalo, F. Sorrentino, e-print `cond-mat/0504335`
15. P. N. McGraw, M. Menzinger, Phys. Rev. E **72**, 015101 (2005)
16. X. Wu, B.-H. Wang, T. Zhou, W.-X. Wang, M. Zhao, H.-J. Yang, Chin. Phys. Lett. **23**, 1046 (2006)
17. H. Hasegawa, Phys. Rev. E **70**, 066107 (2004); H. Hasegawa, Phys. Rev. E **72**, 056139 (2005)

18. M.A. Muñoz, R. Pastor-Satorras, Phys. Rev. Lett. **90**, 204101 (2003)
19. M.A. Muñoz, F. Colaiori, C. Castellano, Phys. Rev. E **72**, 056102 (2005)
20. G. Paul, T. Tanizawa, S. Harvlin, H.E. Stanley, Eur. Phys. J. B **38**, 187 (2004)
21. B. Wang, H.-W. Tang, Z.-L. Xiu, C.-H. Guo, T. Zhou, Physica A **368**, 607 (2006)
22. J.-G. Liu, Z.-T. Wang, Y.-Z. Tang, Mod. Phys. Lett. B **19**, 785 (2005)
23. L. Donetti, P.I. Hurtado, M.A. Muñoz, Phys. Rev. Lett. **95**, 188701 (2005)
24. S. Maslov, K. Sneppen, Science **296**, 910 (2002)
25. B.J. Kim, Phys. Rev. E **69**, 045101 (2004)
26. M. Zhao, T. Zhou, B.-H. Wang, G. Yan, H.-J. Yang, W.-J. Bai, Physica A **371**, 773 (2006)
27. L.M. Pecora, T.L. Carroll, Phys. Rev. Lett. **80**, 2109 (1998)
28. L.M. Pecora, M. Barahona, Chaos Complexity Lett. **1**, 61 (2005)
29. For the details of the memory tabu search algorithm, see, e.g., M.-J. Ji, H.-W. Tang, Applied Mathematics and Computation **159**, 449 (2004); F. Glover, ORSA J. Comput. **1**, 190 (1989); F. Glover, ORSA J. Comput. **2**, 4 (1990)
30. P. Holme, B.J. Kim, Phys. Rev. E **65**, 026107 (2002)
31. P. Erdös, A. Rényi, Publ. Math. **6**, 290 (1959)
32. M.E.J. Newman, Phys. Rev. Lett. **89**, 208701 (2002)
33. S. Pimm, Oikos **35**, 139 (1980)
34. M.E.J. Newman, Phys. Rev. E **64**, 016131 (2001); M.E.J. Newman, Phys. Rev. E **64**, 016132 (2001)
35. T. Zhou, M. Zhao, G.-R. Chen, G. Yan, B.-H. Wang, e-print `cond-mat/0605024`
36. S. Shen-Orr, R. Milo, S. Mangan, U. Alon, Nature Genetics **31**, 64 (2002)
37. R. Milo, S. Shen-Orr, S. Itzkovitz, N. Kashtan, D. Chklovskii, U. Alon, Science **298**, 824 (2002)
38. G. Bianconi, G. Caldarelli, A. Capocci, Phys. Rev. E **71**, 066116 (2005)
39. Y.-Y. Ahn, H. Jeong, B.J. Kim, Physica A **367**, 531 (2006)
40. T. Nishikawa, A.E. Motter, Phys. Rev. E **73**, 065106 (2006)
41. In directed networks, two reverse edges between i and j can be considered as a loop of length two as $i \to j \to i$
42. M. Chavez, D.-U. Hwang, A. Amann, H.G.E. Hentschel, S. Boccaletti, Phys. Rev. Lett. **94**, 218701 (2005)
43. M. Zhao, T. Zhou, B.-H. Wang, Q. Ou, J. Ren, Eur. Phys. J. B **53**, 375 (2006)
44. J. Um, S.-G. Han, B.J. Kim, S.-I. Lee (unpublished)